

# MULTICRITERIA DECISION MAKING

## Business Decision Making

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# THE **STRUCTURE**

Context

1. The context of the topic
  - A. Multi-criteria decision making (MCDM)
  - B. SAW – simple additive weighting

Basics

2. Foundations of the pairwise comparisons method
  - A. Saaty scale
  - B. Transitivity concept

Procedure

3. Pairwise comparisons procedure
  - A. Calculating the weights/priorities
  - B. Calculating the inconsistency in giving judgements

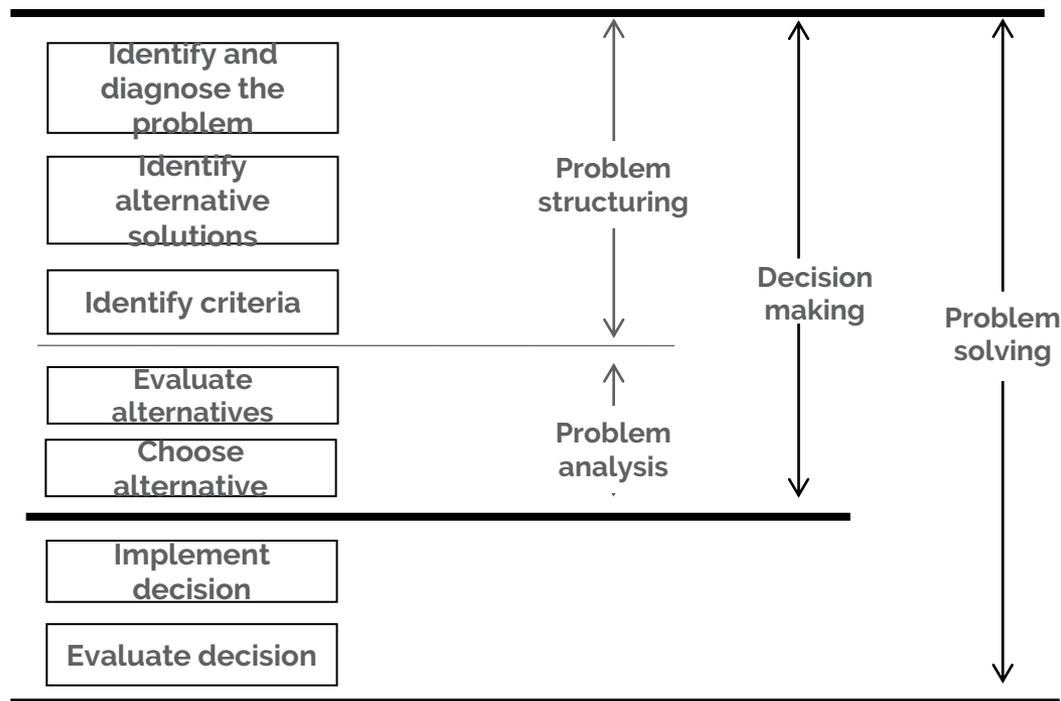
Usage

4. Using the PC procedure
  - A. Methods - AHP
  - B. Applications

# THE **CONTEXT** OF THE TOPIC

- Context
- Basics
- Procedure
- Usage

- PrOACT approach: decomposition of DM problem into elements
  - **Basic elements:** **P**roblem, **O**bjectives (criteria, attributes), **A**lternatives, **C**onsequences and **T**radeoffs
  - **Elements for decision making in turbulent environment:** Risk tolerance, Uncertainty, Linked decisions



# THE **CONTEXT** OF THE TOPIC



- Two decision-making methods groups
  - Methods that support **multicriteria decision making** (basic PrOACT elements)
  - Methods that support **decision making under uncertainty and risk** (PrOACT elements for decision making in turbulent environment)
- Multi-criteria decision-making (MCDM)
  - Decomposition of the main decision-making goal into several sub goals that are described with criteria (attributes)
  - The MCDM problems can be easily described by using the table of values (matrix of decision-making)
  - Alternatives (3), Criteria (3), Consequences/Values (9)

	Education	Experience	CV
Candidate 1	High	5 years	5
Candidate 2	Secondary s	0 years	6
Candidate 3	Secondary s.	2 years	7

# THE **CONTEXT** OF THE TOPIC

Context

- Multi**criteria** decision making is ... about criteria

- Criteria = attributes

- Types of the criteria:

- Qualitative (words): color, design, ...

- Quantitative (numbers): price, weights, height ... two subtypes:

- Min criteria (criteria of costs): price (when we buy), fuel consumption, ...

- Max criteria (criteria of benefits): price (when we sell), quality, ...

Basics

Procedure

Usage

- Types of the criteria 2:

- Natural – price, consumption, ...

- Constructed scale – measuring the properties on some scale

- Proxy criteria – quality of life is measured with GDP



# THE **CONTEXT** OF THE TOPIC

- Context
- Basics
- Procedure
- Usage

- Multi-criteria decision making

	Time	Cost	Satisf.
Make	100	50	High
Buy	10	150	High
SQ	0	0	OK

	Time	Cost	Satisf.	TP
M				
B				
SQ				

- Table of decision making:** alternatives, criteria and consequences
- Methods:** Evenswaps, Electra, Promethee, Topsis, AHP, ANP, **SAW**, Dex method, VIKOR, WINGS, SNAP...
- The **results:**
  - Criteria weights
  - Local priorities of the alternatives per each criterion
  - Total priorities of the alternative – **DECISION!**



# THE **CONTEXT** OF THE TOPIC

- Context
- Basics
- Procedure
- Usage

- Simple additive weighting (SAW)

	Time	Cost	Satisf.	
Make	100	50	High	
Buy	10	150	High	
SQ	0	0	OK	

➔

	Time	Cost	Satisf.	TP
M				
B				
SQ				

- Criteria weights ... 5 procedures
- Local priorities of the alternatives per each criterion ... 7 procedures
- Total priorities of the alternatives ... DECISIONS

$$S_i = w_1 r_{i1} + w_2 r_{i2} + \dots + w_m r_{im} = \sum_{k=1}^m w_k r_{ik}$$

# THE **BASIC FOUNDATIONS** OF THE TOPIC

Context

- **Saaty's scale**

- Founder: prof. Thomas Saaty

- It describes the relation between two elements

- Values of the scale:

- 1 = Two elements are equally important
- 3 = Weak importance of one element over another
- 5 = Strong importance of one element over another
- 7 = Demonstrated importance of one element over another
- 9 = Absolute importance of one element over another
- All real values from scale [1;9] can be used
- Reciprocal values are used when a certain element is dominated by another element



Basics

Procedure

Usage

# THE **BASIC FOUNDATIONS** OF THE TOPIC

Context

- Transitivity concept (math)

Basics



Donald



Boris



Zuzzana

Procedure

$$(D > B \quad \wedge \quad B > Z) \Rightarrow D > Z$$

Usage

# THE **BASIC FOUNDATIONS** OF THE TOPIC

Context

- Transitivity concept (math) + Saaty's scale

Basics



Donald



Boris



Zuzzana

Procedure

$$(D >_3 B \wedge B >_2 Z) \Rightarrow D >_5 Z$$

Usage

IN/CONSISTENCY

What should I do?



# THE **PAIRWISE COMPARISON** PROCEDURE

Context  
Basics  
Procedure  
Usage

- Calculating the criteria weights

	Time	Cost	Satisf.
Make	100	50	High
Buy	10	150	High
SQ	0	0	OK



	Time	Cost	Satisf.	TP
	0.43	0.43	0.14	
M				
B				
SQ				

	T	C	S
T	1	1	3
C	1	1	3
S	1/3	1/3	1
SUM	2.3	2.3	7

0.43	0.43	0.43	0.43
0.43	0.43	0.43	0.43
0.14	0.14	0.14	0.14

**IN/CONSISTENCY**

Input: PC matrix  
Output: CR

CR < 0.1

[Additional reading](#)

What should I do?



# THE **PAIRWISE COMPARISON** PROCEDURE

- Context
- Basics
- Procedure
- Usage

- Calculate the alternatives' priorities (for each column)

	Time	Cost	Satisf.	
<b>Make</b>	100	50	High	
<b>Buy</b>	10	150	High	
<b>SQ</b>	0	0	OK	

➔

	Time	Cost	Satisf.	TP
	0.43	0.43	0.14	
<b>M</b>				
<b>B</b>				
<b>SQ</b>				

Repeat the procedure three times – 3 columns of local priorities!

What should I do?



# THE **PAIRWISE COMPARISON** PROCEDURE

- Context
- Basics
- Procedure
- Usage

- Calculate the alternatives' priorities (for each column)

	Time	Cost	Satisf.	TP
<b>Make</b>	100	50	High	
<b>Buy</b>	10	150	High	
<b>SQ</b>	0	0	OK	

➔

	Time	Cost	Satisf.	TP
	0.43	0.43	0.14	
<b>M</b>				
<b>B</b>				
<b>SQ</b>				

Time	M	B	SQ				
<b>M</b>	1	1/3	1/5	0.11	0.08	0.13	0.11
<b>B</b>	3	1	1/3	0.33	0.23	0.22	0.26
<b>SQ</b>	5	3	1	0.55	0.69	0.65	0.63
<b>SUM</b>	9	4.33	1.53				

What should I do?



# THE **PAIRWISE COMPARISON** PROCEDURE

Context  
Basics  
Procedure  
Usage

- Calculate the alternatives' priorities (for each column)

	Time	Cost	Satisf.	TP
<b>Make</b>	100	50	High	
<b>Buy</b>	10	150	High	
<b>SQ</b>	0	0	OK	

→

	Time	Cost	Satisf.	TP
	0.43	0.43	0.14	
<b>M</b>	0.11			
<b>B</b>	0.26			
<b>SQ</b>	0.63			

Cost	M	B	SQ				
<b>M</b>	1	3	1/2	0.3	0.33	0.29	0.31
<b>B</b>	1/3	1	1/5	0.1	0.11	0.18	0.11
<b>SQ</b>	2	5	1	0.6	0.55	0.58	0.58
<b>SUM</b>	<b>3.33</b>	<b>9</b>	<b>1.7</b>				

What should I do?



# THE PAIRWISE COMPARISON PROCEDURE

Context  
Basics  
Procedure  
Usage

- Calculate the alternatives' priorities (for each column)

	Time	Cost	Satisf.	TP
<b>Make</b>	100	50	High	
<b>Buy</b>	10	150	High	
<b>SQ</b>	0	0	OK	

→

	Time	Cost	Satisf.	TP
	0.43	0.43	0.14	
<b>M</b>	0.11	0.31		
<b>B</b>	0.26	0.11		
<b>SQ</b>	0.63	0.58		

Satis.	M	B	SQ				
<b>M</b>	1	1	4	0.44	0.44	0.44	0.44
<b>B</b>	1	1	4	0.44	0.44	0.44	0.44
<b>SQ</b>	1/4	1/4	1	0.12	0.12	0.12	0.12
<b>SUM</b>	<b>2.25</b>	<b>2.25</b>	<b>9</b>				

What should I do?



# THE **PAIRWISE COMPARISON** PROCEDURE

- Context
- Basics
- Procedure
- Usage

- Aggregating the criteria weights and local priorities in SAW

	Time	Cost	Satisf.
<b>Make</b>	100	50	High
<b>Buy</b>	10	150	High
<b>SQ</b>	0	0	OK



	Time	Cost	Satisf.	TP
	0.43	0.43	0.14	
<b>M</b>	0.11	0.31	0.44	0.24
<b>B</b>	0.26	0.11	0.44	0.22
<b>SQ</b>	0.63	0.58	0.22	0.54

Calculating the total priorities:

$$\xi_i = w_1 r_{i1} + w_2 r_{i2} + \dots + w_m r_{im} = \sum_{k=1}^m w_k r_{ik}$$

# THE **USAGE** OF THE PAIRWISE COMPARISONS

Context

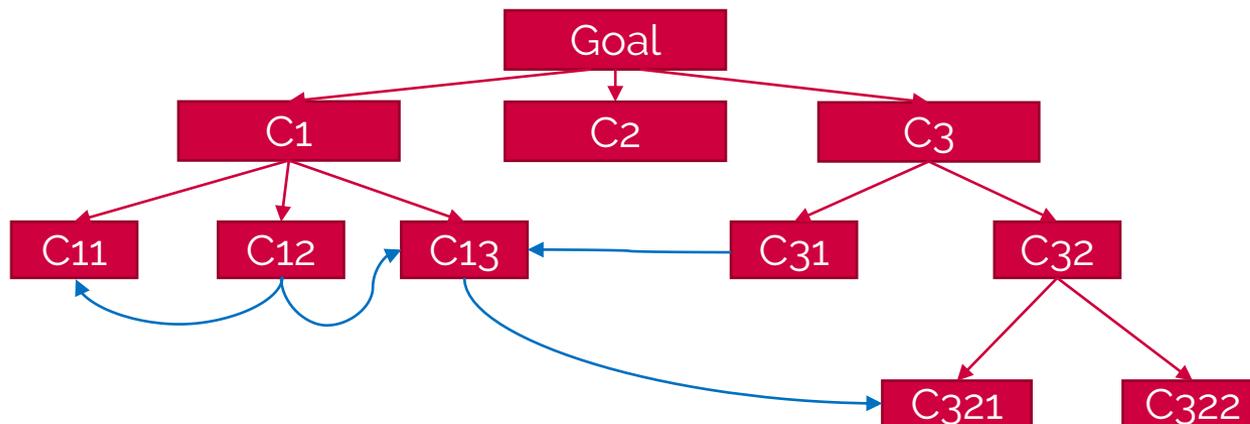
## • Methods

- SIMPLE ADDITIVE WEIGHTING (SAW)
- ANALYTIC HIERARCHY PROCESS (AHP)
- ANALYTIC NETWORK PROCESS (ANP)

Basics

Procedure

Usage



# THE **USAGE** OF THE PAIRWISE COMPARISONS

Context

## • AHP

- The Analytic Hierarchy Process (AHP) (Saaty, 1980) is well known multi-criteria decision-making method
- The AHP is a powerful and flexible decision-making method which helps people to set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered.
- The AHP can combine judgments into a single representative judgment for the group and also including the importance of the individuals themselves.

Basics

Procedure

Usage

# THE **USAGE** OF THE PAIRWISE COMPARISONS

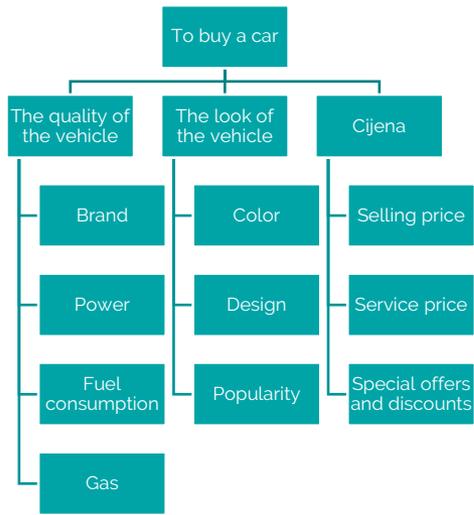
Context  
Basics  
Procedure  
Usage

## • AHP

- It is a more complex variant of the SAW method and PC method – the decision-making problem is more complex
- In the AHP, the criteria are not placed on to one level only – there is a hierarchy structure which is more complex

	Price	Fuel	Color
A1	50000	6	Blue
A2	55000	5	Red
A3	56000	5	Black

	SAW	Price 0,571	FC 0,286	Color 0,143	TP
A1	0,540	0,2	0,163		
A2	0,297	0,4	0,297		
A3	0,163	0,4	0,540		



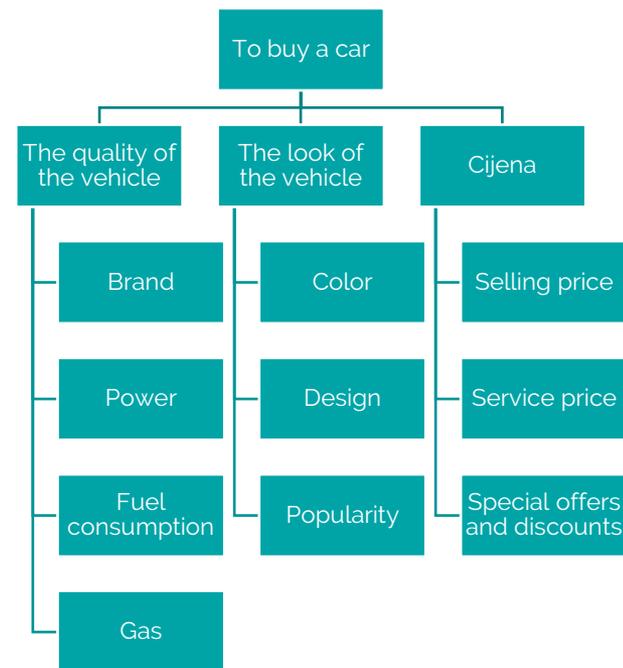
	To buy a car									
	Quality				Look			Price		
	B	P	F	G	C	D	P	SP	SE	D
OC										
Hi30										
P205										

# THE **USAGE** OF THE PAIRWISE COMPARISONS

Context  
Basics  
Procedure  
Usage

## • AHP - steps

1. The AHP enables decision makers to structure decisions hierarchically. The overall goal of the decision is at the top of the model, evaluation criteria in the middle levels, and alternative choices at the bottom
2. Decision makers begin the procedure of pair-wise comparisons on each hierarchy structure level in order to determine the relative importance of elements on each level
3. On the basis of the pair-wise comparisons, relative significance (weights) of elements of the hierarchy structure are calculated (calculation of relative priorities for criteria), which are eventually synthesized into an overall priority list of alternatives (inconsistency)
4. The sensitivity analysis. Sensitivity analysis is used to determine how the priorities of the alternatives change with respect to the importance of the criteria.



	To buy a car									
	Quality				Look			Price		
	B	P	F	G	C	D	P	S	S	D
OC										
Hi30										
P205										

# THE **USAGE** OF THE PAIRWISE COMPARISONS

Context

## • Applications

- Ranking the hospitals in Croatia
- Planning the traffic in Croatia
- Smooth vehicular flow and safe pedestrian crossing separately (Sri Lanka)
- Garage-parking Facility Location Selection in Croatia
- Planning the traffic safety in Turkey
- Selecting the flight procedure design schemes in China
- Prioritisation of the safety control criteria in maritime traffic
- Evaluation Framework for Key Performance Indicators of Railway ITS
- ...

Basics

Procedure

Usage

# ASSIGNMENT 1 PC

- Decision-making table

	Price	Brand	Engine	Fuel consumption	Color
A1	50000	Opel	1.8 diesel	6	Blue
A2	55000	Peugeot	2.2 diesel	5	Red
A3	56000	BMW	2.5 diesel	8	Black

- Calculate the criteria weights using the PC procedure
- Calculate the local priorities of the alternatives using the PC procedure
- Calculate the total priorities of the alternatives

# ASSIGNMENT 2 AHP

1. Choose any strategic MCDM problem you want (it can be personal, business etc.) and describe it.
2. Define criteria relevant for the problem and present them through the hierarchy (at least 2 levels of the criteria, at least 3 criteria at the first level and at least 12 not-decomposed criteria). Describe the criteria.
3. Define at least three alternatives and describe them.
4. Make decision making table
5. Calculate the weights of the criteria, subcriteria and priorities of the alternatives as well as the total priorities. Calculate the inconsistency ratio for each pairwise comparison table.
6. Make the sensitivity analysis changing the weights of each criterion on the first level +/-5% and calculating the total priorities of the alternatives in each change. Make cumulative table. Ex. If you have 3 criteria, you will have 6 sensitivity analyses (each criterion +5%, each criterion -5%). In cumulative table for each sensitivity analysis you have to determine if the firstly ranked alternative stayed first or not.
7. Make final decision.
  - Make word document. 4000 words. You can use Excel to speedup the calculation procedure.